```
In[1]:= (*片持ちはりのべき級数法によるたわみ計算*)

In[2]:= (*5章1節　等分布荷重を受ける片持ちはりの解析*)

In[3]:= Clear["Global`*"]
        クリア

In[4]:= (*級数項の上限，この値を適当に仮定して解析する．*)

In[5]:= nn = 12

Out[5]= 12

In[6]:= (*\psi の級数和の表現式*)

In[7]:= psi = 0;
        Do[psi = psi + a[i] * s^i, {i, 0, nn}]
        反復指定

In[9]:= psi

Out[9]= a[0] + s a[1] + s^2 a[2] + s^3 a[3] + s^4 a[4] + s^5 a[5] +
          s^6 a[6] + s^7 a[7] + s^8 a[8] + s^9 a[9] + s^10 a[10] + s^11 a[11] + s^12 a[12]

In[10]:= (*s ははり先端から測る，境界条件*)

In[11]:= eqbc1 = (psi /. s → 0) - psi0

Out[11]= -psi0 + a[0]

In[12]:= eqbc2 = D[psi, {s, 1}] /. s -> 0
                  微分係数

Out[12]= a[1]

In[13]:= ans1 = Solve[{eqbc1 == 0, eqbc2 == 0}, {a[0], a[1]}][[1]]
                解く

Out[13]= {a[0] → psi0, a[1] → 0}

In[14]:= (*境界条件をpsiに代入し，psiを書き換える．*)

In[15]:= psi = psi /. ans1

Out[15]= psi0 + s^2 a[2] + s^3 a[3] + s^4 a[4] + s^5 a[5] + s^6 a[6] +
          s^7 a[7] + s^8 a[8] + s^9 a[9] + s^10 a[10] + s^11 a[11] + s^12 a[12]

In[16]:= (*Governing equationへ代入．ここで，k^2=P/EI*)

In[17]:= eq1 = D[psi, {s, 2}] + k^2 * Cos[psi]
               微分係数                余弦

Out[17]= 2 a[2] + 6 s a[3] + 12 s^2 a[4] + 20 s^3 a[5] + 30 s^4 a[6] +
          42 s^5 a[7] + 56 s^6 a[8] + 72 s^7 a[9] + 90 s^8 a[10] + 110 s^9 a[11] +
          132 s^10 a[12] + k^2 Cos[psi0 + s^2 a[2] + s^3 a[3] + s^4 a[4] + s^5 a[5] + s^6 a[6] +
             s^7 a[7] + s^8 a[8] + s^9 a[9] + s^10 a[10] + s^11 a[11] + s^12 a[12]]

In[18]:= (*Cos[] の項をsについて級数展開し，式全体をsのべき級数で表現*)
           余弦
```

In[19]:=
```
eq2 = FullSimplify[Series[eq1, {s, 0, nn}]]
      完全に簡約   級数展開
```

Out[19]=

$$\left(2 a[2]+k^2 \mathrm{Cos}[\mathrm{psi0}]\right)+6 a[3] s+\left(12 a[4]-k^2 a[2] \mathrm{Sin}[\mathrm{psi0}]\right) s^2+\left(20 a[5]-k^2 a[3] \mathrm{Sin}[\mathrm{psi0}]\right) s^3+\left(30 a[6]-\frac{1}{2} k^2 \left(a[2]^2 \mathrm{Cos}[\mathrm{psi0}]+2 a[4] \mathrm{Sin}[\mathrm{psi0}]\right)\right) s^4+$$

$$\left(42 a[7]-k^2 (a[2] a[3] \mathrm{Cos}[\mathrm{psi0}]+a[5] \mathrm{Sin}[\mathrm{psi0}])\right) s^5+$$

$$\left(56 a[8]+\frac{1}{6} k^2 \left(-3 \left(a[3]^2+2 a[2] a[4]\right) \mathrm{Cos}[\mathrm{psi0}]+\left(a[2]^3-6 a[6]\right) \mathrm{Sin}[\mathrm{psi0}]\right)\right) s^6+$$

$$\left(72 a[9]+\frac{1}{2} k^2 \left(-2 (a[3] a[4]+a[2] a[5]) \mathrm{Cos}[\mathrm{psi0}]+\left(a[2]^2 a[3]-2 a[7]\right) \mathrm{Sin}[\mathrm{psi0}]\right)\right) s^7+$$

$$\left(90 a[10]+\frac{1}{24} k^2 \left(\left(a[2]^4-12 \left(a[4]^2+2 a[3] a[5]\right)-24 a[2] a[6]\right) \mathrm{Cos}[\mathrm{psi0}]+12 \left(a[2] \left(a[3]^2+a[2] a[4]\right)-2 a[8]\right) \mathrm{Sin}[\mathrm{psi0}]\right)\right) s^8+$$

$$\left(110 a[11]+\frac{1}{6} k^2 \left(\left(-6 a[4] a[5]+a[3] \left(a[2]^3-6 a[6]\right)-6 a[2] a[7]\right) \mathrm{Cos}[\mathrm{psi0}]+\left(a[3]^3+6 a[2] a[3] a[4]+3 a[2]^2 a[5]-6 a[9]\right) \mathrm{Sin}[\mathrm{psi0}]\right)\right) s^9+$$

$$\left(132 a[12]+\frac{1}{120} k^2 \left(10 \left(3 a[2]^2 a[3]^2+2 a[2]^3 a[4]-6 \left(a[5]^2+2 a[4] a[6]+2 a[3] a[7]\right)-12 a[2] a[8]\right) \mathrm{Cos}[\mathrm{psi0}]+\left(-a[2]^5+60 a[2] \left(a[4]^2+2 a[3] a[5]\right)+60 a[2]^2 a[6]+60 \left(a[3]^2 a[4]-2 a[10]\right)\right) \mathrm{Sin}[\mathrm{psi0}]\right)\right) s^{10}+$$

$$\frac{1}{24} k^2 \left(4 \left(3 a[2]^2 a[3] a[4]+a[2]^3 a[5]-6 (a[5] a[6]+a[4] a[7]+a[3] a[8])+a[2] \left(a[3]^3-6 a[9]\right)\right) \mathrm{Cos}[\mathrm{psi0}]+\left(-a[2]^4 a[3]+12 a[3] \left(a[4]^2+a[3] a[5]\right)+24 a[2] (a[4] a[5]+a[3] a[6])+12 a[2]^2 a[7]-24 a[11]\right) \mathrm{Sin}[\mathrm{psi0}]\right) s^{11}-$$

$$\frac{1}{720} \Big(k^2 \Big(\left(a[2]^6-30 a[3]^4-180 a[2]^2 \left(a[4]^2+2 a[3] a[5]\right)-120 a[2]^3 a[6]+360 \left(a[6]^2+2 (a[5] a[7]+a[4] a[8]+a[3] a[9])\right)-360 a[2] \left(a[3]^2 a[4]-2 a[10]\right)\right) \mathrm{Cos}[\mathrm{psi0}]+30 \left(2 a[2]^3 a[3]^2+a[2]^4 a[4]-12 a[2] \left(a[5]^2+2 a[4] a[6]+2 a[3] a[7]\right)-12 a[2]^2 a[8]-4 \left(a[4]^3+6 a[3] a[4] a[5]+3 a[3]^2 a[6]-6 a[12]\right)\right) \mathrm{Sin}[\mathrm{psi0}]\Big)\Big) s^{12}+O[s]^{13}$$

In[20]:=
```
(*この級数の s の係数を見てみる*)
```

In[21]:=
```
eqs0 = Coefficient[eq2, s, 0]
       係数
```

Out[21]= $2 a[2]+k^2 \mathrm{Cos}[\mathrm{psi0}]$

In[22]:=
```
eqs1 = Coefficient[eq2, s, 1]
       係数
```

Out[22]= $6 a[3]$

In[23]:=
```
eqs2 = Coefficient[eq2, s, 2]
       係数
```

Out[23]= $12 a[4]-k^2 a[2] \mathrm{Sin}[\mathrm{psi0}]$

In[24]:=
```
eqs3 = Coefficient[eq2, s, 3]
```
係数

Out[24]= $20\,a[5] - k^2\,a[3]\,\mathrm{Sin}[psi0]$

In[25]:=
```
eqs4 = Coefficient[eq2, s, 4]
```
係数

Out[25]= $30\,a[6] - \frac{1}{2}k^2\left(a[2]^2\,\mathrm{Cos}[psi0] + 2\,a[4]\,\mathrm{Sin}[psi0]\right)$

In[26]:=
```
eqs5 = Coefficient[eq2, s, 5]
```
係数

Out[26]= $42\,a[7] - k^2\,(a[2]\,a[3]\,\mathrm{Cos}[psi0] + a[5]\,\mathrm{Sin}[psi0])$

In[27]:=
```
eqs6 = Coefficient[eq2, s, 6]
```
係数

Out[27]= $56\,a[8] + \frac{1}{6}k^2\left(-3\left(a[3]^2 + 2\,a[2]\,a[4]\right)\mathrm{Cos}[psi0] + \left(a[2]^3 - 6\,a[6]\right)\mathrm{Sin}[psi0]\right)$

In[28]:=
```
Do[
  eqs[i] = Coefficient[eq2, s, i];
  Print[i, " , ", eqs[i]], {i, 0, nn - 2}]
```
反復指定 / 係数 / 出力表示

0 , $2\,a[2] + k^2\,\mathrm{Cos}[psi0]$

1 , $6\,a[3]$

2 , $12\,a[4] - k^2\,a[2]\,\mathrm{Sin}[psi0]$

3 , $20\,a[5] - k^2\,a[3]\,\mathrm{Sin}[psi0]$

4 , $30\,a[6] - \frac{1}{2}k^2\left(a[2]^2\,\mathrm{Cos}[psi0] + 2\,a[4]\,\mathrm{Sin}[psi0]\right)$

5 , $42\,a[7] - k^2\,(a[2]\,a[3]\,\mathrm{Cos}[psi0] + a[5]\,\mathrm{Sin}[psi0])$

6 , $56\,a[8] + \frac{1}{6}k^2\left(-3\left(a[3]^2 + 2\,a[2]\,a[4]\right)\mathrm{Cos}[psi0] + \left(a[2]^3 - 6\,a[6]\right)\mathrm{Sin}[psi0]\right)$

7 , $72\,a[9] + \frac{1}{2}k^2\left(-2\,(a[3]\,a[4] + a[2]\,a[5])\,\mathrm{Cos}[psi0] + \left(a[2]^2\,a[3] - 2\,a[7]\right)\mathrm{Sin}[psi0]\right)$

8 , $90\,a[10] + \frac{1}{24}k^2\left(\left(a[2]^4 - 12\left(a[4]^2 + 2\,a[3]\,a[5]\right) - 24\,a[2]\,a[6]\right)\mathrm{Cos}[psi0] + 12\left(a[2]\left(a[3]^2 + a[2]\,a[4]\right) - 2\,a[8]\right)\mathrm{Sin}[psi0]\right)$

9 , $110\,a[11] + \frac{1}{6}k^2\left(\left(-6\,a[4]\,a[5] + a[3]\left(a[2]^3 - 6\,a[6]\right) - 6\,a[2]\,a[7]\right)\mathrm{Cos}[psi0] + \left(a[3]^3 + 6\,a[2]\,a[3]\,a[4] + 3\,a[2]^2\,a[5] - 6\,a[9]\right)\mathrm{Sin}[psi0]\right)$

10 , $132\,a[12] + \frac{1}{120}k^2\left(10\left(3\,a[2]^2\,a[3]^2 + 2\,a[2]^3\,a[4] - 6\left(a[5]^2 + 2\,a[4]\,a[6] + 2\,a[3]\,a[7]\right) - 12\,a[2]\,a[8]\right)\mathrm{Cos}[psi0] + \left(-a[2]^5 + 60\,a[2]\left(a[4]^2 + 2\,a[3]\,a[5]\right) + 60\,a[2]^2\,a[6] + 60\left(a[3]^2\,a[4] - 2\,a[10]\right)\right)\mathrm{Sin}[psi0]\right)$

In[29]:=
```
(*連立方程式を解くためのリスト作成*)
```

In[30]:=
```
eqlst = {}; anlst = {};
Do[eqlst = Append[eqlst, eqs[i] == 0];
反復指定  追加
 anlst = Append[anlst, a[i + 2]];
          追加
 , {i, 0, nn - 2}]
```

In[32]:=
```
eqlst
```

Out[32]=

$$\Big\{2\,a[2] + k^2 \,\mathrm{Cos}[\mathrm{psi0}] == 0,\; 6\,a[3] == 0,\; 12\,a[4] - k^2\,a[2]\,\mathrm{Sin}[\mathrm{psi0}] == 0,$$

$$20\,a[5] - k^2\,a[3]\,\mathrm{Sin}[\mathrm{psi0}] == 0,\; 30\,a[6] - \frac{1}{2}k^2\left(a[2]^2\,\mathrm{Cos}[\mathrm{psi0}] + 2\,a[4]\,\mathrm{Sin}[\mathrm{psi0}]\right) == 0,$$

$$42\,a[7] - k^2\,(a[2]\,a[3]\,\mathrm{Cos}[\mathrm{psi0}] + a[5]\,\mathrm{Sin}[\mathrm{psi0}]) == 0,$$

$$56\,a[8] + \frac{1}{6}k^2\left(-3\left(a[3]^2 + 2\,a[2]\,a[4]\right)\mathrm{Cos}[\mathrm{psi0}] + \left(a[2]^3 - 6\,a[6]\right)\mathrm{Sin}[\mathrm{psi0}]\right) == 0,$$

$$72\,a[9] + \frac{1}{2}k^2\left(-2\,(a[3]\,a[4] + a[2]\,a[5])\,\mathrm{Cos}[\mathrm{psi0}] + \left(a[2]^2\,a[3] - 2\,a[7]\right)\mathrm{Sin}[\mathrm{psi0}]\right) == 0,$$

$$90\,a[10] + \frac{1}{24}k^2\Big(\left(a[2]^4 - 12\left(a[4]^2 + 2\,a[3]\,a[5]\right) - 24\,a[2]\,a[6]\right)\mathrm{Cos}[\mathrm{psi0}] +$$
$$12\left(a[2]\left(a[3]^2 + a[2]\,a[4]\right) - 2\,a[8]\right)\mathrm{Sin}[\mathrm{psi0}]\Big) == 0,$$

$$110\,a[11] + \frac{1}{6}k^2\Big(\left(-6\,a[4]\,a[5] + a[3]\left(a[2]^3 - 6\,a[6]\right) - 6\,a[2]\,a[7]\right)\mathrm{Cos}[\mathrm{psi0}] +$$
$$\left(a[3]^3 + 6\,a[2]\,a[3]\,a[4] + 3\,a[2]^2\,a[5] - 6\,a[9]\right)\mathrm{Sin}[\mathrm{psi0}]\Big) == 0,$$

$$132\,a[12] + \frac{1}{120}k^2\Big(10\left(3\,a[2]^2\,a[3]^2 + 2\,a[2]^3\,a[4] - 6\left(a[5]^2 + 2\,a[4]\,a[6] + 2\,a[3]\,a[7]\right) -\right.$$
$$\left.12\,a[2]\,a[8]\right)\mathrm{Cos}[\mathrm{psi0}] + \Big(-a[2]^5 + 60\,a[2]\left(a[4]^2 + 2\,a[3]\,a[5]\right) +$$
$$60\,a[2]^2\,a[6] + 60\left(a[3]^2\,a[4] - 2\,a[10]\right)\Big)\mathrm{Sin}[\mathrm{psi0}]\Big) == 0\Big\}$$

In[33]:=
```
anlst
```

Out[33]=
```
{a[2], a[3], a[4], a[5], a[6], a[7], a[8], a[9], a[10], a[11], a[12]}
```

In[34]:=
```
(*a_2,..., a_nn を求めるために連立方程式を解く*)
```

In[35]:=
```
ans1 = Solve[eqlst, anlst][[1]]
           解く
```

Out[35]=

$$\Big\{a[2] \to -\frac{1}{2}k^2\,\mathrm{Cos}[\mathrm{psi0}],\; a[3] \to 0,\; a[4] \to -\frac{1}{24}k^4\,\mathrm{Cos}[\mathrm{psi0}]\,\mathrm{Sin}[\mathrm{psi0}],$$

$$a[5] \to 0,\; a[6] \to \frac{1}{720}k^6\,\mathrm{Cos}[\mathrm{psi0}]\left(3\,\mathrm{Cos}[\mathrm{psi0}]^2 - \mathrm{Sin}[\mathrm{psi0}]^2\right),\; a[7] \to 0,$$

$$a[8] \to -\frac{1}{40320}k^8\,\mathrm{Sin}[\mathrm{psi0}]\left(-33\,\mathrm{Cos}[\mathrm{psi0}]^3 + \mathrm{Cos}[\mathrm{psi0}]\,\mathrm{Sin}[\mathrm{psi0}]^2\right),\; a[9] \to 0,\; a[10] \to$$
$$-\left(\left(k^{10}\,\mathrm{Cos}[\mathrm{psi0}]\left(189\,\mathrm{Cos}[\mathrm{psi0}]^4 - 306\,\mathrm{Cos}[\mathrm{psi0}]^2\,\mathrm{Sin}[\mathrm{psi0}]^2 + \mathrm{Sin}[\mathrm{psi0}]^4\right)\right)\Big/ 3628800\right),$$

$$a[11] \to 0,\; a[12] \to -\frac{1}{479001600}$$
$$k^{12}\,\mathrm{Sin}[\mathrm{psi0}]\left(8289\,\mathrm{Cos}[\mathrm{psi0}]^5 - 2766\,\mathrm{Cos}[\mathrm{psi0}]^3\,\mathrm{Sin}[\mathrm{psi0}]^2 + \mathrm{Cos}[\mathrm{psi0}]\,\mathrm{Sin}[\mathrm{psi0}]^4\right)\Big\}$$

In[36]:=
```
psians = FullSimplify[psi /. ans1]
```
完全に簡約

Out[36]=

$$\text{psi0} - \frac{1}{2} k^2 s^2 \text{Cos}[\text{psi0}] + \frac{1}{720} k^6 s^6 (2\,\text{Cos}[\text{psi0}] + \text{Cos}[3\,\text{psi0}]) - \frac{1}{3\,628\,800} k^{10} s^{10} \text{Cos}[\text{psi0}] (33 + 94\,\text{Cos}[2\,\text{psi0}] + 62\,\text{Cos}[4\,\text{psi0}]) - \frac{1}{24} k^4 s^4 \text{Cos}[\text{psi0}]\,\text{Sin}[\text{psi0}] + \frac{1}{80\,640} k^8 s^8 (49\,\text{Cos}[\text{psi0}] + 17\,\text{Cos}[3\,\text{psi0}])\,\text{Sin}[\text{psi0}] - \left(k^{12} s^{12} (4835\,\text{Cos}[\text{psi0}] + 2763\,\text{Cos}[3\,\text{psi0}] + 691\,\text{Cos}[5\,\text{psi0}])\,\text{Sin}[\text{psi0}]\right) / 479\,001\,600$$

In[37]:=
```
(*無次元変数 s1=s/L, s=L でたわみ角ゼロ*)
```

In[38]:=
```
eq2 = (psians /. {s → s1 * L}) /. s1 → 1
```

Out[38]=

$$\text{psi0} - \frac{1}{2} k^2 L^2 \text{Cos}[\text{psi0}] + \frac{1}{720} k^6 L^6 (2\,\text{Cos}[\text{psi0}] + \text{Cos}[3\,\text{psi0}]) - \frac{1}{3\,628\,800} k^{10} L^{10} \text{Cos}[\text{psi0}] (33 + 94\,\text{Cos}[2\,\text{psi0}] + 62\,\text{Cos}[4\,\text{psi0}]) - \frac{1}{24} k^4 L^4 \text{Cos}[\text{psi0}]\,\text{Sin}[\text{psi0}] + \frac{1}{80\,640} k^8 L^8 (49\,\text{Cos}[\text{psi0}] + 17\,\text{Cos}[3\,\text{psi0}])\,\text{Sin}[\text{psi0}] - \left(k^{12} L^{12} (4835\,\text{Cos}[\text{psi0}] + 2763\,\text{Cos}[3\,\text{psi0}] + 691\,\text{Cos}[5\,\text{psi0}])\,\text{Sin}[\text{psi0}]\right) / 479\,001\,600$$

In[39]:=
```
(*k^2=P/EI => k^2L^2=PL^2/EI => k^2L^2→P1*)
```

In[40]:=
```
(*P1→ 無次元荷重*)
```

In[41]:=
```
nonlin1 = eq2 /. {k → P1^(1/2)/L}
```

Out[41]=

$$\text{psi0} - \frac{1}{2} \text{P1}\,\text{Cos}[\text{psi0}] + \frac{1}{720} \text{P1}^3 (2\,\text{Cos}[\text{psi0}] + \text{Cos}[3\,\text{psi0}]) - \frac{1}{3\,628\,800} \text{P1}^5 \text{Cos}[\text{psi0}] (33 + 94\,\text{Cos}[2\,\text{psi0}] + 62\,\text{Cos}[4\,\text{psi0}]) - \frac{1}{24} \text{P1}^2 \text{Cos}[\text{psi0}]\,\text{Sin}[\text{psi0}] + \frac{1}{80\,640} \text{P1}^4 (49\,\text{Cos}[\text{psi0}] + 17\,\text{Cos}[3\,\text{psi0}])\,\text{Sin}[\text{psi0}] - \left(\text{P1}^6 (4835\,\text{Cos}[\text{psi0}] + 2763\,\text{Cos}[3\,\text{psi0}] + 691\,\text{Cos}[5\,\text{psi0}])\,\text{Sin}[\text{psi0}]\right) / 479\,001\,600$$

In[42]:=
```
(*psi0 と P1の関係を求める*)
```

In[43]:=
```
Print["psi0    P1"];
```
出力表示
```
Do[
```
反復指定
```
 anspsi2 = FindRoot[nonlin1 /. {psi0 → i * Pi / 180}, {P1, 1}][[1]][[2]];
```
根を求める　円周率
```
 Print[i, " , ", anspsi2], {i, 20, 80, 20}]
```
出力表示

```
psi0    P1
20 , 0.730592
40 , 1.69231
60 , 3.40621
80 , 8.66843
```

In[45]:=
```
(*参考のために楕円積分解も表示*)
```

In[46]:=
```
(*psi0   p         q^2      delta/L   ( L-Delta)/L)
0,0.707107,8.88178*10^-16,7.45058*10^-9,0.
5,0.737277,0.175021,0.0581376,0.99797
10,0.766044,0.353003,0.116035,0.991884
15,0.793353,0.537053,0.173455,0.981758
20,0.819152,0.730592,0.230164,0.967617
25,0.843391,0.937551,0.285935,0.949493
30,0.866025,1.16264,0.340551,0.927422
35,0.887011,1.41171,0.393808,0.901442
40,0.906308,1.6923,0.445517,0.871586
45,0.92388,2.01447,0.495511,0.837871
50,0.939693,2.39217,0.543651,0.800287
55,0.953717,2.84559,0.589836,0.758772
60,0.965926,3.40541,0.63402,0.713174
65,0.976296,4.12138,0.676242,0.66318
70,0.984808,5.0812,0.716688,0.60817
75,0.991445,6.45953,0.755834,0.546873
80,0.996195,8.67877,0.794849,0.476389
85,0.999048,13.2333,0.837329,0.38802   *)
```

In[47]:=
```
(*たわみ計算*)
```

In[48]:=
```
(* y=Sin[psi_0]*Integrate[Cos[beta],{s,0,s}]+
      正弦            積分           余弦
  Cos[psi_0]*Integrate[Sin[beta],{s,0,s}]*)
  余弦            積分           正弦
```

In[49]:=
```
(* x=Cos[psi_0]*Integrate[Cos[beta],{s,0,s}]-
      余弦            積分           余弦
  Sin[psi_0]*Integrate[Sin[beta],{s,0,s}]*)
  正弦            積分           正弦
```

In[50]:=
```
(*Here, beta=a_2*s^2+a3*s^3+....*)
  現在地
```

In[51]:=
```
beta1 = psi - psi0
```

Out[51]= $s^2 a[2] + s^3 a[3] + s^4 a[4] + s^5 a[5] + s^6 a[6] + s^7 a[7] + s^8 a[8] + s^9 a[9] + s^{10} a[10] + s^{11} a[11] + s^{12} a[12]$

In[52]:=
```
Series[Cos[beta1], {s, 0, nn - 1}]
```
級数展開 余弦

Out[52]=

$$1-\frac{1}{2}a[2]^2 s^4 - a[2]\,a[3]\,s^5 + \left(-\frac{1}{2}a[3]^2 - a[2]\,a[4]\right)s^6 + \left(-a[3]\,a[4] - a[2]\,a[5]\right)s^7 + \left(\frac{a[2]^4}{24} - \frac{a[4]^2}{2} - a[3]\,a[5] - a[2]\,a[6]\right)s^8 + \left(\frac{1}{6}a[2]^3 a[3] - a[4]\,a[5] - a[3]\,a[6] - a[2]\,a[7]\right)s^9 + \left(\frac{1}{4}a[2]^2 a[3]^2 + \frac{1}{6}a[2]^3 a[4] - \frac{a[5]^2}{2} - a[4]\,a[6] - a[3]\,a[7] - a[2]\,a[8]\right)s^{10} + \left(\frac{1}{6}a[2]\,a[3]^3 + \frac{1}{2}a[2]^2 a[3]\,a[4] + \frac{1}{6}a[2]^3 a[5] - a[5]\,a[6] - a[4]\,a[7] - a[3]\,a[8] - a[2]\,a[9]\right)s^{11} + O[s]^{12}$$

In[53]:=
```
(*級数展開して，項別に積分する*)
```

In[54]:=
```
T1 = FullSimplify[Integrate[Series[Cos[beta1], {s, 0, nn}], {s, 0, s}]]
```
完全に簡約 積分 級数展開 余弦

Out[54]=

$$s - \frac{1}{10}s^5 a[2]^2 - \frac{1}{6}s^6 a[2]\,a[3] - \frac{1}{14}s^7\left(a[3]^2 + 2\,a[2]\,a[4]\right) - \frac{1}{8}s^8\left(a[3]\,a[4] + a[2]\,a[5]\right) + \frac{1}{216}s^9\left(a[2]^4 - 12\left(a[4]^2 + 2\,a[3]\,a[5]\right) - 24\,a[2]\,a[6]\right) + \frac{1}{60}s^{10}\left(-6\,a[4]\,a[5] + a[3]\left(a[2]^3 - 6\,a[6]\right) - 6\,a[2]\,a[7]\right) + \frac{1}{132}s^{11}\left(3\,a[2]^2 a[3]^2 + 2\,a[2]^3 a[4] - 6\left(a[5]^2 + 2\,a[4]\,a[6] + 2\,a[3]\,a[7]\right) - 12\,a[2]\,a[8]\right) + \frac{1}{72}s^{12}\left(3\,a[2]^2 a[3]\,a[4] + a[2]^3 a[5] - 6\left(a[5]\,a[6] + a[4]\,a[7] + a[3]\,a[8]\right) + a[2]\left(a[3]^3 - 6\,a[9]\right)\right) - \frac{1}{9360}s^{13}\left(a[2]^6 - 30\,a[3]^4 - 180\,a[2]^2\left(a[4]^2 + 2\,a[3]\,a[5]\right) - 120\,a[2]^3 a[6] + 360\left(a[6]^2 + 2\left(a[5]\,a[7] + a[4]\,a[8] + a[3]\,a[9]\right)\right) - 360\,a[2]\left(a[3]^2 a[4] - 2\,a[10]\right)\right)$$

In[55]:=
```
Do[Print[i, " , ", Coefficient[T1, s, i]], {i, 1, nn}]
```
⋯ 出力表示 係数

1 , 1

2 , 0

3 , 0

4 , 0

5 , $-\frac{1}{10}\,a[2]^2$

6 , $-\frac{1}{6}\,a[2]\,a[3]$

7 , $\frac{1}{14}\left(-a[3]^2-2\,a[2]\,a[4]\right)$

8 , $\frac{1}{8}\,(-a[3]\,a[4]-a[2]\,a[5])$

9 , $\frac{1}{216}\left(a[2]^4-12\left(a[4]^2+2\,a[3]\,a[5]\right)-24\,a[2]\,a[6]\right)$

10 , $\frac{1}{60}\left(-6\,a[4]\,a[5]+a[3]\left(a[2]^3-6\,a[6]\right)-6\,a[2]\,a[7]\right)$

11 , $\frac{1}{132}\left(3\,a[2]^2\,a[3]^2+2\,a[2]^3\,a[4]-6\left(a[5]^2+2\,a[4]\,a[6]+2\,a[3]\,a[7]\right)-12\,a[2]\,a[8]\right)$

12 ,
$\frac{1}{72}\left(3\,a[2]^2\,a[3]\,a[4]+a[2]^3\,a[5]-6\,(a[5]\,a[6]+a[4]\,a[7]+a[3]\,a[8])+a[2]\left(a[3]^3-6\,a[9]\right)\right)$

In[56]:=
```
T2 = FullSimplify[Integrate[Series[Sin[beta1], {s, 0, nn}], {s, 0, s}]]
```
完全に簡約 積分 級数展開 正弦

Out[56]=
$$\frac{1}{3}s^3\,a[2]+\frac{1}{4}s^4\,a[3]+\frac{1}{5}s^5\,a[4]+\frac{1}{6}s^6\,a[5]-\frac{1}{42}s^7\left(a[2]^3-6\,a[6]\right)-$$
$$\frac{1}{16}s^8\left(a[2]^2\,a[3]-2\,a[7]\right)-\frac{1}{18}s^9\left(a[2]\left(a[3]^2+a[2]\,a[4]\right)-2\,a[8]\right)-$$
$$\frac{1}{60}s^{10}\left(a[3]^3+6\,a[2]\,a[3]\,a[4]+3\,a[2]^2\,a[5]-6\,a[9]\right)+\frac{1}{1320}$$
$$s^{11}\left(a[2]^5-60\,a[2]\left(a[4]^2+2\,a[3]\,a[5]\right)-60\,a[2]^2\,a[6]-60\left(a[3]^2\,a[4]-2\,a[10]\right)\right)+$$
$$\frac{1}{288}s^{12}\left(a[2]^4\,a[3]-12\,a[3]\left(a[4]^2+a[3]\,a[5]\right)-\right.$$
$$\left.24\,a[2]\,(a[4]\,a[5]+a[3]\,a[6])-12\,a[2]^2\,a[7]+24\,a[11]\right)+$$
$$\frac{1}{312}s^{13}\left(2\,a[2]^3\,a[3]^2+a[2]^4\,a[4]-12\,a[2]\left(a[5]^2+2\,a[4]\,a[6]+2\,a[3]\,a[7]\right)-\right.$$
$$\left.12\,a[2]^2\,a[8]-4\left(a[4]^3+6\,a[3]\,a[4]\,a[5]+3\,a[3]^2\,a[6]-6\,a[12]\right)\right)$$

In[57]:=
```
Do[Print[i, " , ", Coefficient[T2, s, i]], {i, 1, nn}]
```
… 出力表示 係数

1 , 0

2 , 0

3 , $\frac{a[2]}{3}$

4 , $\frac{a[3]}{4}$

5 , $\frac{a[4]}{5}$

6 , $\frac{a[5]}{6}$

7 , $\frac{1}{42}\left(-a[2]^3 + 6\,a[6]\right)$

8 , $\frac{1}{16}\left(-a[2]^2\,a[3] + 2\,a[7]\right)$

9 , $\frac{1}{18}\left(-a[2]\left(a[3]^2 + a[2]\,a[4]\right) + 2\,a[8]\right)$

10 , $\frac{1}{60}\left(-a[3]^3 - 6\,a[2]\,a[3]\,a[4] - 3\,a[2]^2\,a[5] + 6\,a[9]\right)$

11 , $\frac{1}{1320}\left(a[2]^5 - 60\,a[2]\left(a[4]^2 + 2\,a[3]\,a[5]\right) - 60\,a[2]^2\,a[6] - 60\left(a[3]^2\,a[4] - 2\,a[10]\right)\right)$

12 , $\frac{1}{288}\left(a[2]^4\,a[3] - 12\,a[3]\left(a[4]^2 + a[3]\,a[5]\right) - 24\,a[2]\,(a[4]\,a[5] + a[3]\,a[6]) - 12\,a[2]^2\,a[7] + 24\,a[11]\right)$

In[58]:= (*たわみの計算*)

In[59]:= defy1 = T1 * Sin[psi0] + T2 * Cos[psi0]
正弦 余弦

Out[59]=
$$\left(\frac{1}{3} s^3 a[2] + \frac{1}{4} s^4 a[3] + \frac{1}{5} s^5 a[4] + \frac{1}{6} s^6 a[5] - \frac{1}{42} s^7 \left(a[2]^3 - 6 a[6]\right) - \frac{1}{16} s^8 \left(a[2]^2 a[3] - 2 a[7]\right) - \frac{1}{18} s^9 \left(a[2] \left(a[3]^2 + a[2] a[4]\right) - 2 a[8]\right) - \frac{1}{60} s^{10} \left(a[3]^3 + 6 a[2] a[3] a[4] + 3 a[2]^2 a[5] - 6 a[9]\right) + \frac{1}{1320} s^{11} \left(a[2]^5 - 60 a[2] \left(a[4]^2 + 2 a[3] a[5]\right) - 60 a[2]^2 a[6] - 60 \left(a[3]^2 a[4] - 2 a[10]\right)\right) + \frac{1}{288} s^{12} \left(a[2]^4 a[3] - 12 a[3] \left(a[4]^2 + a[3] a[5]\right) - 24 a[2] (a[4] a[5] + a[3] a[6]) - 12 a[2]^2 a[7] + 24 a[11]\right) + \frac{1}{312} s^{13} \left(2 a[2]^3 a[3]^2 + a[2]^4 a[4] - 12 a[2] \left(a[5]^2 + 2 a[4] a[6] + 2 a[3] a[7]\right) - 12 a[2]^2 a[8] - 4 \left(a[4]^3 + 6 a[3] a[4] a[5] + 3 a[3]^2 a[6] - 6 a[12]\right)\right)\right) Cos[psi0] +$$

$$\left(s - \frac{1}{10} s^5 a[2]^2 - \frac{1}{6} s^6 a[2] a[3] - \frac{1}{14} s^7 \left(a[3]^2 + 2 a[2] a[4]\right) - \frac{1}{8} s^8 (a[3] a[4] + a[2] a[5]) + \frac{1}{216} s^9 \left(a[2]^4 - 12 \left(a[4]^2 + 2 a[3] a[5]\right) - 24 a[2] a[6]\right) + \frac{1}{60} s^{10} \left(-6 a[4] a[5] + a[3] \left(a[2]^3 - 6 a[6]\right) - 6 a[2] a[7]\right) + \frac{1}{132} s^{11} \left(3 a[2]^2 a[3]^2 + 2 a[2]^3 a[4] - 6 \left(a[5]^2 + 2 a[4] a[6] + 2 a[3] a[7]\right) - 12 a[2] a[8]\right) + \frac{1}{72} s^{12} \left(3 a[2]^2 a[3] a[4] + a[2]^3 a[5] - 6 (a[5] a[6] + a[4] a[7] + a[3] a[8]) + a[2] \left(a[3]^3 - 6 a[9]\right)\right) - \frac{1}{9360} s^{13} \left(a[2]^6 - 30 a[3]^4 - 180 a[2]^2 \left(a[4]^2 + 2 a[3] a[5]\right) - 120 a[2]^3 a[6] + 360 \left(a[6]^2 + 2 (a[5] a[7] + a[4] a[8] + a[3] a[9])\right) - 360 a[2] \left(a[3]^2 a[4] - 2 a[10]\right)\right)\right) Sin[psi0]$$

In[60]:=
```
defx1 = T1 * Cos[psi0] - T2 * Sin[psi0]
```
余弦 正弦

Out[60]=

$$\begin{aligned}&\Big(s-\frac{1}{10}s^5a[2]^2-\frac{1}{6}s^6a[2]a[3]-\frac{1}{14}s^7\left(a[3]^2+2a[2]a[4]\right)-\\&\frac{1}{8}s^8(a[3]a[4]+a[2]a[5])+\frac{1}{216}s^9\left(a[2]^4-12\left(a[4]^2+2a[3]a[5]\right)-24a[2]a[6]\right)+\\&\frac{1}{60}s^{10}\left(-6a[4]a[5]+a[3]\left(a[2]^3-6a[6]\right)-6a[2]a[7]\right)+\frac{1}{132}s^{11}\\&\left(3a[2]^2a[3]^2+2a[2]^3a[4]-6\left(a[5]^2+2a[4]a[6]+2a[3]a[7]\right)-12a[2]a[8]\right)+\\&\frac{1}{72}s^{12}\Big(3a[2]^2a[3]a[4]+a[2]^3a[5]-6(a[5]a[6]+a[4]a[7]+a[3]a[8])+\\&\quad a[2]\left(a[3]^3-6a[9]\right)\Big)-\frac{1}{9360}\\&s^{13}\Big(a[2]^6-30a[3]^4-180a[2]^2\left(a[4]^2+2a[3]a[5]\right)-120a[2]^3a[6]+360\\&\quad\left(a[6]^2+2(a[5]a[7]+a[4]a[8]+a[3]a[9])\right)-360a[2]\left(a[3]^2a[4]-2a[10]\right)\Big)\Big)\\&\text{Cos}[psi0]-\Big(\frac{1}{3}s^3a[2]+\frac{1}{4}s^4a[3]+\frac{1}{5}s^5a[4]+\frac{1}{6}s^6a[5]-\frac{1}{42}s^7\left(a[2]^3-6a[6]\right)-\\&\frac{1}{16}s^8\left(a[2]^2a[3]-2a[7]\right)-\frac{1}{18}s^9\left(a[2]\left(a[3]^2+a[2]a[4]\right)-2a[8]\right)-\\&\frac{1}{60}s^{10}\left(a[3]^3+6a[2]a[3]a[4]+3a[2]^2a[5]-6a[9]\right)+\frac{1}{1320}\\&s^{11}\left(a[2]^5-60a[2]\left(a[4]^2+2a[3]a[5]\right)-60a[2]^2a[6]-60\left(a[3]^2a[4]-2a[10]\right)\right)+\\&\frac{1}{288}s^{12}\Big(a[2]^4a[3]-12a[3]\left(a[4]^2+a[3]a[5]\right)-\\&\quad 24a[2](a[4]a[5]+a[3]a[6])-12a[2]^2a[7]+24a[11]\Big)+\\&\frac{1}{312}s^{13}\Big(2a[2]^3a[3]^2+a[2]^4a[4]-12a[2]\left(a[5]^2+2a[4]a[6]+2a[3]a[7]\right)-\\&\quad 12a[2]^2a[8]-4\left(a[4]^3+6a[3]a[4]a[5]+3a[3]^2a[6]-6a[12]\right)\Big)\Big)\ \text{Sin}[psi0]\end{aligned}$$

In[61]:=
```
(*たわみの無次元化*)
```

In[62]:=
```
defy2 = FullSimplify[((defy1 /. ans1) /. {s → L * s1}) / L]
```
完全に簡約

Out[62]=

$$\begin{aligned}&\frac{1}{97\,297\,200}\Big(-27k^{10}L^{10}s1^{11}\,\text{Cos}[psi0]^6\left(273+127k^2L^2s1^2\,\text{Sin}[psi0]\right)+\\&2106k^6L^6s1^7\,\text{Cos}[psi0]^4\left(165+k^2L^2s1^2\,\text{Sin}[psi0]\left(55+9k^2L^2s1^2\,\text{Sin}[psi0]\right)\right)+\\&4290\,s1\left(22\,680\,\text{Sin}[psi0]-k^6L^6s1^6\left(18+k^2L^2s1^2\,\text{Sin}[psi0]\right)\text{Sin}[2\,psi0]^2\right)-\\&8k^2L^2s1^3\,\text{Cos}[psi0]^2\Big(2\,027\,025+k^2L^2s1^2\,\text{Sin}[psi0]\\&\quad\left(405\,405+2k^6L^6s1^6\,\text{Sin}[psi0]^3\left(39+k^2L^2s1^2\,\text{Sin}[psi0]\right)-63k^8L^8s1^8\,\text{Sin}[2\,psi0]^2\right)\Big)\Big)\end{aligned}$$

In[63]:= **defy3 = defy2 /. {k → P1^(1/2)/L}**

Out[63]= $\frac{1}{97\,297\,200}\left(-27\,P1^5\,s1^{11}\,\text{Cos}[psi0]^6\,\left(273+127\,P1\,s1^2\,\text{Sin}[psi0]\right)+2106\,P1^3\,s1^7\,\text{Cos}[psi0]^4\,\left(165+P1\,s1^2\,\text{Sin}[psi0]\,\left(55+9\,P1\,s1^2\,\text{Sin}[psi0]\right)\right)+4290\,s1\,\left(22\,680\,\text{Sin}[psi0]-P1^3\,s1^6\,\left(18+P1\,s1^2\,\text{Sin}[psi0]\right)\,\text{Sin}[2\,psi0]^2\right)-8\,P1\,s1^3\,\text{Cos}[psi0]^2\,\left(2\,027\,025+P1\,s1^2\,\text{Sin}[psi0]\,\left(405\,405+2\,P1^3\,s1^6\,\text{Sin}[psi0]^3\,\left(39+P1\,s1^2\,\text{Sin}[psi0]\right)-63\,P1^4\,s1^8\,\text{Sin}[2\,psi0]^2\right)\right)\right)$

In[64]:= **defx2 = FullSimplify[((defx1 /. ans1) /. {s → L * s1}) / L]**

完全に簡約

Out[64]= $\frac{1}{6\,227\,020\,800}\left(-128\,s1\,\left(-48\,648\,600+k^4\,L^4\,s1^4\,\left(810\,810-10\,725\,k^4\,L^4\,s1^4+146\,k^8\,L^8\,s1^8\right)\right)\,\text{Cos}[psi0]-1584\,k^4\,L^4\,s1^5\,\left(32\,760-845\,k^4\,L^4\,s1^4+16\,k^8\,L^8\,s1^8\right)\,\text{Cos}[3\,psi0]+518\,918\,400\,k^2\,L^2\,s1^3\,\text{Sin}[2\,psi0]+k^6\,L^6\,s1^7\,\left(2\,k^2\,L^2\,s1^2\,\left(265\,980-9557\,k^4\,L^4\,s1^4\right)\,\text{Cos}[5\,psi0]-5461\,k^6\,L^6\,s1^6\,\text{Cos}[7\,psi0]-308\,880\,(32\,\text{Sin}[2\,psi0]+17\,\text{Sin}[4\,psi0])+78\,k^4\,L^4\,s1^4\,(2072\,(\text{Sin}[2\,psi0]+\text{Sin}[4\,psi0])+691\,\text{Sin}[6\,psi0])\right)\right)$

In[65]:= **defx3 = defx2 /. {k → P1^(1/2)/L}**

Out[65]= $\frac{1}{6\,227\,020\,800}\left(-128\,s1\,\left(-48\,648\,600+P1^2\,s1^4\,\left(810\,810-10\,725\,P1^2\,s1^4+146\,P1^4\,s1^8\right)\right)\,\text{Cos}[psi0]-1584\,P1^2\,s1^5\,\left(32\,760-845\,P1^2\,s1^4+16\,P1^4\,s1^8\right)\,\text{Cos}[3\,psi0]+518\,918\,400\,P1\,s1^3\,\text{Sin}[2\,psi0]+P1^3\,s1^7\,\left(2\,P1\,s1^2\,\left(265\,980-9557\,P1^2\,s1^4\right)\,\text{Cos}[5\,psi0]-5461\,P1^3\,s1^6\,\text{Cos}[7\,psi0]-308\,880\,(32\,\text{Sin}[2\,psi0]+17\,\text{Sin}[4\,psi0])+78\,P1^2\,s1^4\,(2072\,(\text{Sin}[2\,psi0]+\text{Sin}[4\,psi0])+691\,\text{Sin}[6\,psi0])\right)\right)$

In[66]:= **(* Deflections of the beam *)**

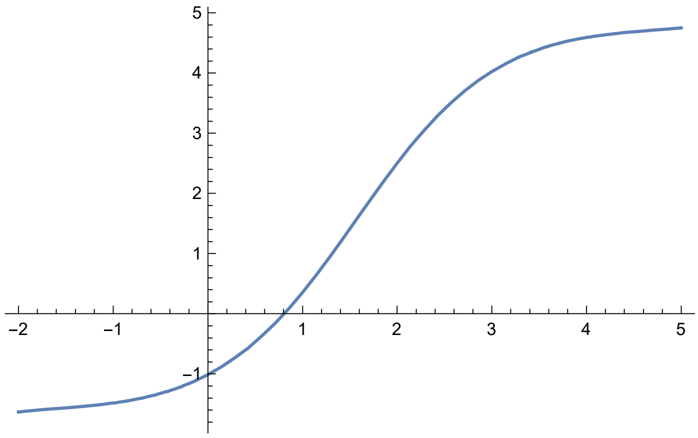
In[67]:= **(*Preparation of relation between psi0 and P1 *)**

In[68]:= **nonlin1**

Out[68]= $psi0-\frac{1}{2}\,P1\,\text{Cos}[psi0]+\frac{1}{720}\,P1^3\,(2\,\text{Cos}[psi0]+\text{Cos}[3\,psi0])-\frac{1}{3\,628\,800}P1^5\,\text{Cos}[psi0]\,(33+94\,\text{Cos}[2\,psi0]+62\,\text{Cos}[4\,psi0])-\frac{1}{24}\,P1^2\,\text{Cos}[psi0]\,\text{Sin}[psi0]+\frac{1}{80\,640}P1^4\,(49\,\text{Cos}[psi0]+17\,\text{Cos}[3\,psi0])\,\text{Sin}[psi0]-\left(P1^6\,(4835\,\text{Cos}[psi0]+2763\,\text{Cos}[3\,psi0]+691\,\text{Cos}[5\,psi0])\,\text{Sin}[psi0]\right)/479\,001\,600$

In[69]:=
```
Plot[nonlin1 /. P1 → 120 * Pi / 180, {psi0, -2, 5}]
プロット                    円周率
```

Out[69]=



In[70]:=
```
FindRoot[nonlin1 /. psi0 → 20 * Pi / 180, {P1, 1}][[1]][[2]]
根を求める                        円周率
```

Out[70]= 0.730592

In[71]:=
```
(*psi0 を与えてP1(無次元荷重)を求める*)
```

In[72]:=
```
Print["psi0     P1      y/L        x/L "];
出力表示
deflstx = {};
deflsty = {};
Do[
反復指定
  psi00 = i * 10 * Pi / 180;
                 円周率
  ansP1 = FindRoot[(nonlin1 /. {psi0 → psi00}) == 0, {P1, 1}];(*Print[];
         根を求める                                            出力表示
   Print["alpha=",i*10,", w1=", ansP1[[1]][[2]]]*);
   出力表示
   defx4 = (defx3 /. ansP1) /. {s1 → 1, psi0 → psi00};
   defy4 = (defy3 /. ansP1) /. {s1 → 1, psi0 → psi00};
   deflstx = Append[deflstx, {defx4, ansP1[[1]][[2]]}];
             追加
   deflsty = Append[deflsty, {defy4, ansP1[[1]][[2]]}];
             追加
 Print[i * 10, " , ", ansP1[[1]][[2]], " , ", defy4, " , ", defx4];
 出力表示
 , {i, 1, 8}]
(*結果のプロット*)
ListPlot[{deflstx, deflsty}, Frame → True, GridLines → Automatic, Joined → False,
リストプロット                 枠      真      格子線      自動        点の結合   偽
 PlotStyle → PointSize[0.02], FrameLabel → {"Deflections", "Load (Pl^2/EI)"}]
             ポイントサイズ       枠ラベル
```

```
psi0    P1        y/L        x/L
10 , 0.353003 , 0.116035 , 0.991884
20 , 0.730592 , 0.230164 , 0.967617
30 , 1.16264 , 0.340552 , 0.927424
40 , 1.69231 , 0.44553 , 0.871599
50 , 2.39231 , 0.543734 , 0.800296
60 , 3.40621 , 0.634253 , 0.712942
70 , 5.08352 , 0.716522 , 0.606783
80 , 8.66843 , 0.790532 , 0.473719
```

Out[76]=